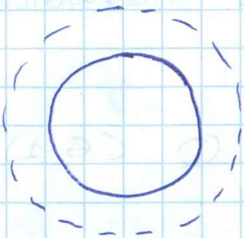
## 6.1



$$I = \oint_S \vec{j}\, d\vec{S} = \oint_S \lambda \frac{\vec{D}}{\varepsilon\varepsilon_0}\, d\vec{S} =$$

$$= \frac{\lambda}{\varepsilon\varepsilon_0} Q_1 = \frac{\lambda}{\varepsilon\varepsilon_0} C_1 U_1$$

$$I = -\frac{\lambda}{\varepsilon\varepsilon_0} Q_2 = -\frac{\lambda}{\varepsilon\varepsilon_0} C_2 U_2$$

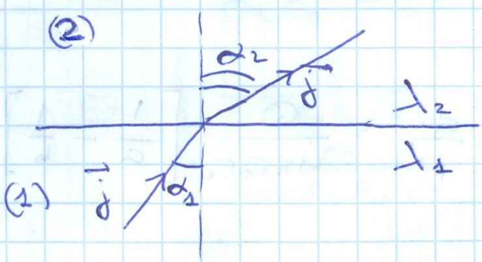
$$I\varepsilon\varepsilon_0 = \lambda C_1 U_1 = -\lambda C_2 U_2$$

$$\frac{U_1}{U_2} = -\frac{C_2}{C_1} = -\frac{4\pi\varepsilon_0 R_2}{4\pi\varepsilon_0 R_1} = -\frac{R_2}{R_1}$$

$$U = U_1 - U_2 = -U_2\left(1 + \frac{R_2}{R_1}\right)$$

$$U_2 = -\frac{U R_1}{R_1 + R_2} \qquad U_1 = \frac{U R_2}{R_1 + R_2}$$

## 6.2




1) $E_{\tau 1} = E_{\tau 2}$

$E_1 \sin\alpha_1 = E_2 \sin\alpha_2$

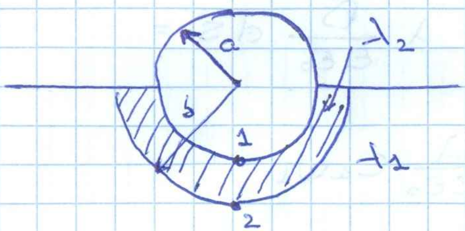
2) из непрерывности тока следует:

$$j_1 \cos\alpha_1 = j_2 \cos\alpha_2$$

$$\lambda_1 E_1 \cos\alpha_1 = \lambda_2 E_2 \cos\alpha_2$$

$$\frac{\operatorname{tg}\alpha_1}{\lambda_1} = \frac{\operatorname{tg}\alpha_2}{\lambda_2} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{\operatorname{tg}\alpha_2}{\operatorname{tg}\alpha_1}$$

6.3




1) Погрузим шар полностью в землю:

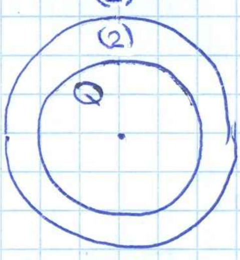
$$I = \oint_S \bar{j}\, d\bar{S} = \frac{\lambda}{\varepsilon \varepsilon_0} Q \quad (6.1)$$

$$I = \frac{\lambda_1}{\varepsilon_1 \varepsilon_0} Q = \frac{\lambda_2}{\varepsilon_2 \varepsilon_0} Q$$

$$U = \overbrace{(\varphi_1 - \varphi_2)}^{U_2} + \overbrace{(\varphi_2 - \varphi_3)}^{U_1} = I\hat{R} = I(R_1 + R_2) =$$

$$= IR_1 + IR_2 = U_1 + U_2$$

$$U_1 = IR_1 = \frac{\lambda_1}{\varepsilon_1 \varepsilon_0} Q R_1 \qquad U_2 = \frac{\lambda_2}{\varepsilon_2 \varepsilon_0} Q R_2$$




$$E = \begin{cases} \dfrac{Q}{4\pi \varepsilon_0 \varepsilon_1 r^2}, & \text{область (1)} \\ \dfrac{Q}{4\pi \varepsilon_0 \varepsilon_2 r^2}, & \text{область (2)} \end{cases}$$

$$U_2 = \int_a^b \bar{E}\, d\bar{r} = \frac{Q}{4\pi \varepsilon_0 \varepsilon_2} \int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi \varepsilon_0 \varepsilon_2} \left[\frac{1}{a} - \frac{1}{b}\right]$$

$$U_1 = \int_b^\infty \bar{E}\, d\bar{r} = \frac{Q}{4\pi \varepsilon_0 \varepsilon_1} \frac{1}{b}$$

$$\frac{Q}{4\pi \varepsilon_0 \varepsilon_2} \left[\frac{1}{a} - \frac{1}{b}\right] = \frac{\lambda_2}{\varepsilon_2 \varepsilon_0} Q R_2 \Rightarrow R_2 = \frac{1}{4\pi \lambda_2} \left[\frac{1}{a} - \frac{1}{b}\right]$$

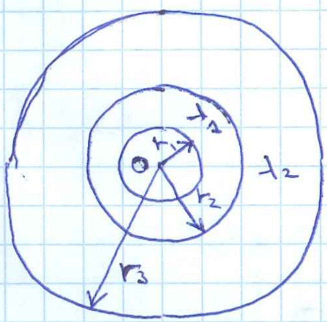
$$\frac{Q}{4\pi\varepsilon_0\varepsilon_1}\frac{1}{b} = \frac{\lambda_1}{\varepsilon_0\varepsilon_1} Q R_1 \Rightarrow R_1 = \frac{1}{4\pi\lambda_1}\cdot\frac{1}{b}$$

$$\hat{R} = R_1 + R_2 = \frac{1}{4\pi\lambda_2}\left[\frac{1}{a} - \frac{1}{b}\right] + \frac{1}{4\pi\lambda_1 b}$$

г) тогда для шара погружённого наполовину:

$$R = 2\hat{R} = \frac{1}{2\pi\lambda_2}\left[\frac{1}{a} - \frac{1}{b}\right] + \frac{1}{2\pi\lambda_1 b}$$

$\boxed{6.10}$ аналогично 6.3




$$I = \frac{\lambda_1}{\varepsilon_1\varepsilon_0} Q = \frac{\lambda_2}{\varepsilon_2\varepsilon_0} Q$$

$$U = \underbrace{(\varphi_1 - \varphi_2)}_{U_1} + \underbrace{(\varphi_2 - \varphi_3)}_{U_2} =$$

$$= I R_1 + I R_2$$

$$U_1 = \frac{\lambda_1}{\varepsilon_1\varepsilon_0} Q R_1 \qquad U_2 = \frac{\lambda_2}{\varepsilon_2\varepsilon_0} Q R_2$$

$$E = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0\varepsilon_1 r^2}, & r \in (r_1, r_2) \\ \dfrac{Q}{4\pi\varepsilon_0\varepsilon_2 r^2}, & r \in (r_2, r_3) \end{cases}$$

$$U_1 = \int_{r_1}^{r_2} \bar{E}\, d\bar{r} = \frac{Q}{4\pi\varepsilon_0\varepsilon_1}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$U_2 = \int_{r_2}^{r_3} \bar{E}\, d\bar{r} = \frac{Q}{4\pi\varepsilon_0\varepsilon_2}\left[\frac{1}{r_2} - \frac{1}{r_3}\right]$$

$$R_1 = \frac{1}{4\pi\lambda_1}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$R_2 = \frac{1}{4\pi\lambda_2}\left[\frac{1}{r_2} - \frac{1}{r_3}\right]$$

$$\cancel{N_1 = \frac{U^2}{R_1} = \frac{4\pi\lambda_1 U^2}{r_2 - r_1} \cdot r_2 r_1}$$

$$\cancel{N_2 = \frac{U^2}{R_2} = \frac{4\pi\lambda_2 U^2}{r_3 - r_2} \cdot r_3 r_2}$$

$$N_1 = I^2 R_1 = \frac{U^2 R_1}{(R_1 + R_2)^2}$$

$$N_2 = I^2 R_2 = \frac{U^2 R_2}{(R_1 + R_2)^2}$$

$\boxed{6.11}$ аналогично 6.10.

из 5.5 известно:

$$E = \begin{cases} \dfrac{\sigma r_1}{\varepsilon_1 \varepsilon_0 r}, & r_1 < r < r_2 \\ \dfrac{\sigma r_1}{\varepsilon_2 \varepsilon_0 r}, & r_2 < r < r_3 \end{cases}$$

$$U_1 = \int_{r_1}^{r_2} \bar{E}\, d\bar{r} = \frac{\sigma r_1}{\varepsilon_1 \varepsilon_0} \ln\frac{r_2}{r_1}$$

$$U_2 = \int_{r_2}^{r_3} \bar{E}\, d\bar{r} = \frac{\sigma r_1}{\varepsilon_2 \varepsilon_0} \ln\frac{r_3}{r_1}$$

$$U_1 = \frac{\lambda_1}{\varepsilon_1 \varepsilon_0} Q R_1 = \frac{\lambda_1}{\varepsilon_1 \varepsilon_0} \sigma \cdot 2\pi r_1 l \cdot R_1$$

$$U_2 = \frac{\lambda_2}{\varepsilon_2 \varepsilon_0} \sigma \cdot 2\pi r_2 l \cdot R_2$$

$$\frac{\lambda_1}{\cancel{\varepsilon_1 \varepsilon_0}} \cancel{\sigma}\, 2\pi \cancel{r_1}\, l\, R_1 = \cancel{\sigma r_1} \frac{1}{\cancel{\varepsilon_0 \varepsilon_1}} \ln \frac{r_2}{r_1}$$

$$R_1 = \frac{1}{2\pi \lambda_1 l} \ln \frac{r_2}{r_1} \quad ; \quad R_2 = \frac{1}{2\pi \lambda_2 l} \ln \frac{r_3}{r_2}$$

$$N_1 = I^2 R_1 = \frac{U^2 R_1}{(R_1 + R_2)^2} \qquad N_2 = \frac{U^2 R_2}{(R_1 + R_2)^2}$$

$\boxed{6.3}$ $$I = \oint_S \vec{j}\, d\vec{S} = \frac{\lambda}{\varepsilon \varepsilon_0} Q_1 = -\frac{\lambda}{\varepsilon \varepsilon_0} Q_2 \quad (6.1)$$

$$U = \varphi_1 - \varphi_2 = \frac{Q_1}{C_1} - \frac{Q_2}{C_2} = \frac{Q_1}{C_1} + \frac{Q_1}{C_2} = $$
$$= Q_1 \left( \frac{1}{C_1} + \frac{1}{C_2} \right)$$

$$U = IR = \frac{\lambda}{\varepsilon \varepsilon_0} Q_1 R$$

$$R = \frac{\varepsilon \varepsilon_0}{\lambda} \left( \frac{1}{C_1} + \frac{1}{C_2} \right)$$

$\boxed{6.4}$ из задачи 6.3 известно:

$$\hat{R} = \frac{\varepsilon \varepsilon_0}{\lambda} \left( \frac{1}{C_1} + \frac{1}{C_2} \right)$$

тогда для шаров полностью утоплен. в земле

$$\hat{R} = \frac{\varepsilon \varepsilon_0}{\lambda} \left( \frac{1}{4\pi \varepsilon \varepsilon_0 a_1} + \frac{1}{4\pi \varepsilon \varepsilon_0 a_2} \right) = \frac{1}{4\pi \lambda} \left( \frac{1}{a_1} + \frac{1}{a_2} \right)$$

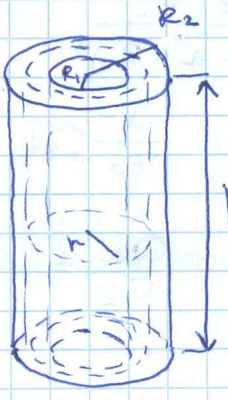
тогда для шаров погружённых наполовину:

$$R = 2\tilde{R} = \frac{1}{2\pi\lambda}\left(\frac{1}{a_1} + \frac{1}{a_2}\right)$$

$\boxed{6.8}$ из задачи 6.9 известно:

$$R = \frac{\varepsilon\varepsilon_0}{\lambda}\left(\frac{1}{4\pi\varepsilon\varepsilon_0 r} + \frac{1}{4\pi\varepsilon\varepsilon_0 r}\right) = \frac{1}{2\pi\lambda r}$$

$\boxed{6.9}$




т. Гаусса:

$$\oint_S D_1\, ds = Q$$

$$D_1 \cdot 2\pi r \cdot \cancel{\ell} = \sigma \cdot \cancel{2\pi} R_1 \cdot \cancel{\ell}$$

$$E_1 = \frac{\sigma R_1}{\varepsilon_1 \varepsilon_0 r}$$

$$U_1 = \varphi_1 - \varphi_2 = \int_{R_1}^{R_2} E\, d\bar{r} = \frac{\sigma R_1}{\varepsilon_1\varepsilon_0}\ln\frac{R_2}{R_1}$$

$$C_1 = \frac{Q}{U} = 2\pi \cancel{R_1} \cdot h\, \cancel{\sigma} \cdot \frac{\varepsilon_1\varepsilon_0}{\cancel{\sigma R_1} \ln\frac{R_2}{R_1}} =$$

$$= \frac{2\pi\varepsilon_1\varepsilon_0 h}{\ln\frac{R_2}{R_1}}$$

$$N_1 = \frac{U^2}{R_1} = \left\{ R_1 = \frac{\varepsilon_1\varepsilon_0}{\lambda_1 C_1}\ (*) \right\} = \frac{\lambda_1 C_1}{\varepsilon_1\varepsilon_0} U^2 =$$

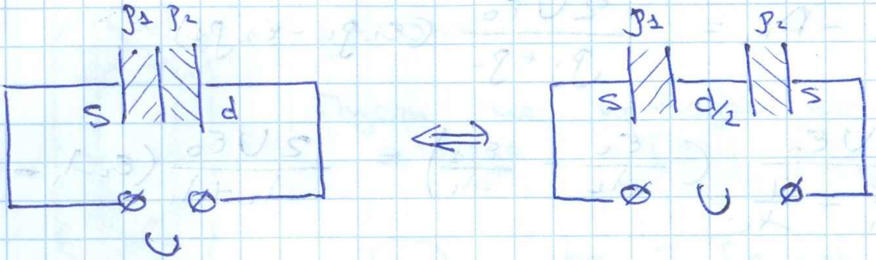
$$N_1 = \frac{2\pi \lambda_1 h}{\ln \frac{R_2}{R_1}} U^2 \qquad N_2 = \frac{2\pi \lambda_2 h}{\ln \frac{R_2}{R_1}} U^2$$

$$\frac{N_1}{N_2} = \frac{\lambda_1}{\lambda_2}$$

$$(*): \quad I = \oint_S \vec{j}\, d\vec{S} = \frac{\lambda}{\varepsilon\varepsilon_0} Q = \frac{\lambda}{\varepsilon\varepsilon_0} CU$$

$$U = IR \Rightarrow R = \frac{\varepsilon\varepsilon_0}{\lambda C}$$

$\boxed{6.6}$

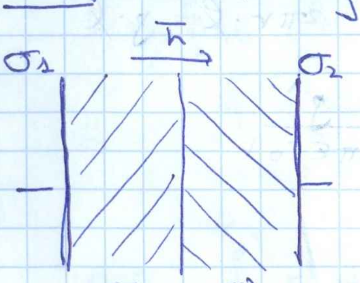



$U \equiv \mathcal{E}_0$

$$R_1 = \rho_1 \frac{d}{2S}; \qquad R_2 = \rho_2 \frac{d}{2S}$$

$$N_1 = I^2 R_1 = \frac{U^2 R_1}{(R_1+R_2)^2} \qquad N_2 = \frac{U^2 R_2}{(R_1+R_2)^2}$$

$\boxed{6.7}$ $\quad \lambda_i = \frac{1}{\rho_i}$




Граничное условие для границы:

$D_{n_2} - D_{n_1} = \sigma$ (убрали диэлектр)

$D_{n_1} = \varepsilon_0 \varepsilon_1 E_1$

$D_{n_2} = \varepsilon_0 \varepsilon_2 E_2$

Граничное условие для пластин:

$$\sigma_1 = D_1 = D_{n_1} \qquad \sigma_2 = -D_2 = -D_{n_2}$$

$\vec{j} = const$ $\quad \vec{j} = \lambda \vec{E}$ $\quad j = \frac{I}{S} = \frac{U}{R_1+R_2}\cdot\frac{1}{S}$

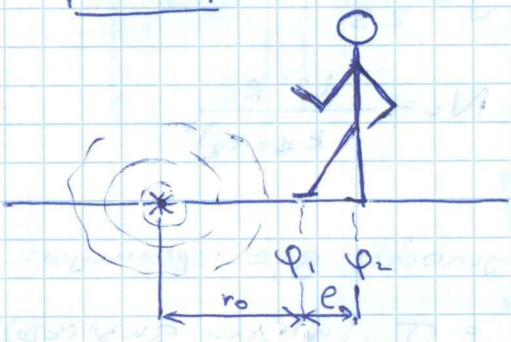
$$E_1 = \frac{j}{\lambda_1} = \frac{I}{\lambda_1 S} = \frac{U}{R_1+R_2}\frac{1}{\lambda_1 S} = \frac{2U\rho_1}{(\rho_1+\rho_2)d}$$

$$E_2 = \frac{j}{\lambda_2} = \frac{2U\rho_2}{(\rho_1+\rho_2)d}$$

$$\sigma = D_{n_2} - D_{n_1} = \frac{2U\varepsilon_0}{\rho_1+\rho_2}(\varepsilon_2\rho_2 - \varepsilon_1\rho_1) =$$

$$= \frac{2U\varepsilon_0}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}\left(\frac{\varepsilon_2}{\lambda_2}-\frac{\varepsilon_1}{\lambda_1}\right) = \frac{2U\varepsilon_0}{\lambda_2+\lambda_1}(\varepsilon_2\lambda_1 - \varepsilon_1\lambda_2)$$

$\boxed{6.12}$




$$U_{\text{шаг}} = \varphi_1 - \varphi_2$$


$$\oint D\,ds = Q$$

$$\varepsilon\varepsilon_0 E \cdot 2\pi r \cdot \ell = \rho \cdot \ell$$

$$E = \frac{\rho}{2\pi\varepsilon\varepsilon_0 r}$$

$$j = \lambda E = \frac{\lambda\rho}{2\pi\varepsilon\varepsilon_0 r}$$

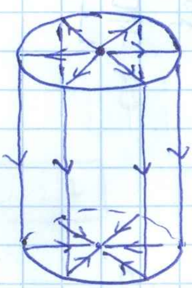
$$I = \oint_S \vec{j}\,d\vec{s} = \underbrace{\frac{\lambda\rho}{2\pi\varepsilon\varepsilon_0 r}}_{j}\cdot\underbrace{\frac{2\pi r\cdot L}{2}}_{S} = \frac{\lambda\rho L}{2\varepsilon\varepsilon_0} \Rightarrow$$

$$\Rightarrow \rho = \frac{2I\varepsilon\varepsilon_0}{\lambda L}$$

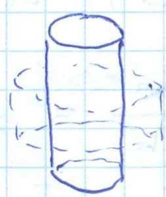
$$E = \frac{1}{\cancel{2}\pi \cancel{\varepsilon\varepsilon_0} r} \cdot \frac{\cancel{2} I \cancel{\varepsilon\varepsilon_0}}{\lambda L} = \frac{I}{\pi \lambda L \cdot r}$$

$$U_{max} = \varphi_1 - \varphi_2 = \int_{r_0}^{r_0+l} E\,dr = \frac{I}{\pi\lambda L} \ln\left(1 + \frac{l}{r_0}\right)$$

$\boxed{6.93}$



Пусть на проводах есть плотность заряда $\rho \left[\frac{Кл}{м}\right]$, тогда



по т. Гаусса:

$$E = \frac{\rho}{2\pi\varepsilon\varepsilon_0 r}$$

$$j = \lambda E = \frac{\rho\lambda}{2\pi\varepsilon\varepsilon_0 r}$$

$$I = \oint \vec{j}\, d\vec{S} = \frac{\rho\lambda}{\cancel{2\pi}\varepsilon\varepsilon_0 \cancel{r}} \cdot \cancel{2\pi r} \cdot \delta = \frac{\rho\lambda\delta}{\varepsilon\varepsilon_0}$$

$$E = \frac{I}{2\pi\cancel{\varepsilon}\varepsilon_0 r} \cdot \frac{\varepsilon\varepsilon_0}{\lambda\delta} = \frac{I}{2\pi\lambda\delta r}$$

$$\Delta\varphi = \int_{d/2}^{D/2} E\,dr = \frac{I}{2\pi\lambda\delta} \ln\frac{D}{d}$$

$$R_{торц.} = \frac{\Delta\varphi}{I} = \frac{1}{2\pi\lambda\delta} \ln\frac{D}{d}$$

из 6.5 известно:

$$R_{бок} = \frac{\varepsilon\varepsilon_0}{\lambda C_{бок}} = \left\{ C_{бок} = \frac{\varepsilon\varepsilon_0 S}{d} = \frac{\varepsilon\varepsilon_0 \pi D \delta}{\ell} \right\} =$$

$$= \frac{\ell}{\lambda \pi D \delta}$$

$$R = 2R_{торц} + R_{бок} = \frac{2}{2\pi\lambda\delta} \ln\frac{D}{d} + \frac{\ell}{\lambda\pi D\delta} =$$

$$= \frac{1}{\pi\lambda\delta}\left[\frac{\ell}{D} + \ln\frac{D}{d}\right]$$

$\boxed{6.16}$ $\lambda(x) = \frac{k}{x^2}$

$$I = const = \oint_S \bar{j}\, d\bar{s} = j(x)\cdot 2\pi x \cdot \ell$$

$$E(x) = \frac{j(x)}{\lambda(x)} = \frac{I}{2\pi x\cdot \ell}\cdot\frac{x^2}{k} = \frac{I}{2\pi \ell k}\, x$$

$$U_0 = \int_{R_1}^{R_2} E(x)\, d\bar{x} = \frac{I}{4\pi \ell k}\cdot\left[R_2^2 - R_1^2\right]$$

$$E(x) = \frac{I}{2\pi\ell\cdot k}\, x = \frac{x}{\cancel{2\pi\ell\cdot k}}\cdot\frac{\overset{2}{\cancel{4\pi\ell k}}}{R_2^2 - R_1^2}\, U_0 =$$

$$= \frac{2U_0 x}{R_2^2 - R_1^2}$$

$$D(x) = \varepsilon\varepsilon_0 E(x) = \varepsilon\varepsilon_0 \frac{2U_0 x}{R_2^2 - R_1^2}$$

$$\rho(x) = \operatorname{div} D^{(x)} = \frac{6U_0\varepsilon\varepsilon_0}{R_2^2 - R_1^2}$$ (свободные заряды)

$$\operatorname{div} D = \frac{1}{x^2}\frac{\partial}{\partial x}\left(D(x)x^2\right) = \frac{1}{x^2}\frac{\partial}{\partial x}\left(\frac{2U_0\varepsilon\varepsilon_0}{R_2^2-R_1^2}x^3\right) =$$

$$= \frac{1}{x^2}\cdot\frac{6U\varepsilon\varepsilon_0}{R_2^2-R_1^2}x^2 = \frac{6U\varepsilon\varepsilon_0}{R_2^2-R_1^2}$$

Другой способ посчитать $\rho(x)$ –
– уравнение Пуассона

$$\Delta\varphi = -\frac{\rho}{\varepsilon\varepsilon_0} = \frac{1}{x^2}\frac{\partial}{\partial x}\left(x^2\frac{\partial\varphi}{\partial x}\right) = -\frac{1}{x^2}\frac{\partial}{\partial x}\left(x^2 E(x)\right)$$

$$\rho = \frac{6U\varepsilon\varepsilon_0}{R_2^2-R_1^2}$$

**6.17** $\quad \mathcal{D} = \lambda(x)\cdot E^2(x) = \text{const}$

$$I = \oint_S \bar{j}\,d\bar{S} = j(x)\cdot 4\pi x^2 = \text{const}$$

$$E(x) = \frac{j(x)}{\lambda(x)} = \frac{I}{4\pi x^2\lambda(x)}$$

$$\mathcal{D} = \frac{I^2}{16\pi^2 x^4\lambda(x)} = \text{const} \Rightarrow \lambda(x) = \frac{k}{x^4}$$

**6.18** $\quad I = \oint_S \bar{j}\,d\bar{S} = j(x)\cdot 2\pi x\cdot \ell = \text{const}$

$$E(x) = \frac{j(x)}{\lambda(x)} = \frac{I}{2\pi x\ell}\cdot\frac{1}{\lambda(x)} = \text{const} \Rightarrow$$

$$\Rightarrow \lambda(x) = c_1 j(x) = c_2\cdot\frac{1}{x}$$

$\boxed{6.19}$

$$\begin{cases} \lambda(x) = \alpha x + \beta \\ \lambda(0) = \lambda_1 \\ \lambda(l) = \lambda_2 \end{cases} \Rightarrow \quad \beta = \lambda_1 \qquad \alpha = \frac{\lambda_2 - \lambda_1}{l}$$

$$E(x) = \frac{j}{\lambda(x)} = \frac{j}{\alpha x + \beta}$$

$$-\frac{\rho}{\varepsilon\varepsilon_0} = \Delta\varphi = \frac{\partial}{\partial^2 x}\varphi(x) = -\frac{\partial E(x)}{\partial x} = \frac{\alpha j}{(\alpha x + \beta)^2}$$

$$\rho = -\frac{\alpha\varepsilon\varepsilon_0 j}{(\alpha x + \beta)^2} = -\frac{\lambda_2 - \lambda_1}{l} \cdot \frac{\varepsilon\varepsilon_0 j}{\left(\frac{\lambda_2 - \lambda_1}{l} + \lambda_1\right)^2} =$$

$$= \frac{\varepsilon\varepsilon_0 j (\lambda_1 - \lambda_2) l}{((\lambda_2 - \lambda_1)x + l\lambda_1)^2}$$

$\boxed{6.20}$ $$I_i = \frac{\lambda}{\varepsilon\varepsilon_0} q_i \quad (6.1)$$

$$N_i = I_i U_i = I_i \varphi_i = \frac{\lambda}{\varepsilon\varepsilon_0} q_i \varphi_i$$

$$N = \sum_{i=1}^{n} \frac{\lambda}{\varepsilon\varepsilon_0} q_i \varphi_i = \frac{\lambda}{\varepsilon\varepsilon_0} \sum_{i=1}^{n} q_i \varphi_i$$

$\boxed{6.21}$ $$R = \frac{\varepsilon\varepsilon_0}{\lambda C} \qquad N = \frac{U_0^2}{R} = \frac{U_0^2 \lambda C}{\varepsilon\varepsilon_0} =$$

$$= \frac{U_0^2 \lambda}{\cancel{\varepsilon\varepsilon_0}} \cdot 4\pi \cancel{\varepsilon\varepsilon_0} \frac{R_1 R_2}{R_2 - R_1} = \frac{4\pi U_0^2 \lambda}{\frac{1}{R_1} - \frac{1}{R_2}}$$

$\boxed{6.22}$

Ток разрядки: $I = -\dfrac{dq}{dt}$

с другой стороны: $I = \dfrac{U}{R} = \dfrac{q}{RC} = \dfrac{\lambda q}{\varepsilon\varepsilon_0}$

$$\frac{dq}{dt} + \frac{\lambda q}{\varepsilon\varepsilon_0} = 0$$

$$\frac{dq}{q} = -\frac{\lambda}{\varepsilon\varepsilon_0}dt$$

$$q(t) = q_0 \exp\left(-\frac{\lambda}{\varepsilon\varepsilon_0}t\right) \quad ; \quad q_0 = U_0 C$$

$$I(t) = \frac{\lambda q(t)}{\varepsilon\varepsilon_0} = \lambda \cdot U_0 \cdot \frac{2\pi\cancel{\varepsilon\varepsilon_0}}{\cancel{\varepsilon\varepsilon_0}} \frac{\ell}{\ln R_2/R_1} \exp\left(-\frac{\lambda}{\varepsilon\varepsilon_0}t\right) =$$

$$= \frac{2\pi\lambda \ell U_0}{\ln R_2/R_1} \exp\left(-\frac{\lambda}{\varepsilon\varepsilon_0}t\right)$$

(*): емкость такого конденсатора (6.5)

$\boxed{6.23}$ $\quad C(t) = \dfrac{\varepsilon\varepsilon_0 S}{vt + d_0} \quad ; \quad q(t) = U_0 C(t) \quad \varepsilon = 1$

$$I_0 = \frac{dq}{dt} = \frac{d}{dt}\left(U_0 \frac{\varepsilon\varepsilon_0 S}{vt + d_0}\right) = -\frac{v\varepsilon\varepsilon_0 S U_0}{(vt + d_0)^2}$$

$$\Delta q = q_2 - q_1 = C_2 U_0 - C_1 U_0 = \varepsilon\varepsilon_0 S U_0 \left(\frac{1}{vt + d_0} - \frac{1}{d_0}\right) < 0$$

$\Rightarrow$ отрицат. заряд набегает на пластину конденсатора $\Rightarrow$

$\Rightarrow \quad I = -I_0$.

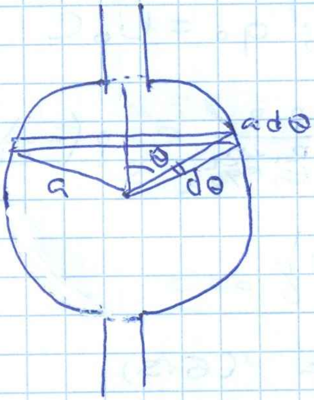
$\boxed{6.24}$ $C(t) = \frac{\varepsilon\varepsilon_0 \alpha t^2}{d}$ ; $q = U_0 C(t)$

$I_0 = \frac{dq}{dt} = U_0 \frac{2\varepsilon\varepsilon_0 \alpha t}{d}$

$\Delta q = q_2 - q_1 = U_0 \cdot (C_2 - C_1) = \frac{\varepsilon\varepsilon_0\alpha}{d}(t_2^2 - t_1^2) > 0$

$I = I_0.$

$\boxed{6.15}$




т.к. $\delta \ll a$, то сферу можно представить в виде цилиндров.

из задачи 6.13 известно, что

$R_{бок} = R = \frac{l}{4\pi d \delta}$

тогда $dR = \frac{\cancel{a} d\theta}{\lambda \cdot 2\pi \cancel{a} \sin\theta \, \delta}$

$R_0 = \int\limits_{\theta_0}^{\pi-\theta_0} \frac{d\theta}{\lambda \cdot 2\pi \sin\theta \, \delta} = \frac{1}{2\pi\lambda\delta} \ln \mathrm{tg}\frac{\theta}{2}\Big|_{\theta_0}^{\pi-\theta_0} =$

$= \frac{1}{2\pi\lambda\delta}\left[\ln \mathrm{tg}\left(\frac{\pi}{2} - \frac{\theta_0}{2}\right) - \ln \mathrm{tg}\frac{\theta_0}{2}\right] =$

$= -\frac{1}{\pi\lambda\delta} \ln \mathrm{tg}\frac{\theta_0}{2}$ ; $\mathrm{tg}\frac{\theta_0}{2} < 1$

$\theta_0 < \frac{\pi}{2}$

$\ln\left(\mathrm{tg}\frac{\theta_0}{2}\right) < 0.$

$\boxed{6.14}$

$$I = const = \frac{U}{R_0} = \frac{U \pi \lambda \delta}{\ln ctg \frac{\Theta_0}{2}} \quad (6.15)$$

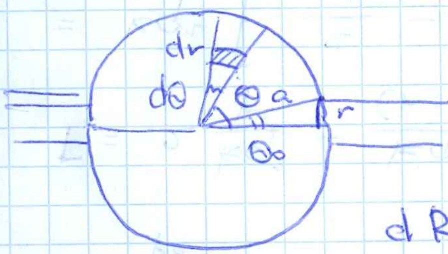
пусть $\varphi(A) = 0$, тогда:

$$\varphi(\Theta) - \varphi(A) = I R(\Theta) = I \int_{\Theta_0}^{\Theta} dR =$$

$$= \frac{U \cancel{\pi \lambda \delta}}{\ln ctg \frac{\Theta_0}{2}} \cdot \frac{1}{2\cancel{\pi \lambda \delta}} \ln tg \frac{\Theta}{2} \Big|_{\Theta_0}^{\Theta} =$$

$$= U \frac{\ln tg \frac{\Theta}{2} - \ln tg \frac{\Theta_0}{2}}{-2 \ln tg \frac{\Theta_0}{2}}$$

$\boxed{6.25}$



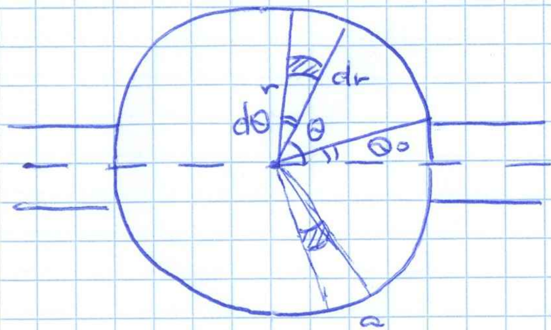

Шар можно представить в виде мн-ва концентр. сфер толщины dr.

$$dR = -\frac{1}{\pi \lambda dr} \ln tg \frac{\Theta_0}{2} \quad (6.15).$$

$$A = \int_0^a \frac{1}{dR} = -\frac{\pi \lambda}{\ln tg \frac{\Theta_0}{2}} \int_0^a dr = -\frac{\pi \lambda a}{\ln tg \frac{\Theta_0}{2}} \Rightarrow$$

$$\Rightarrow R = \frac{1}{A} = -\frac{\ln tg \frac{\Theta_0}{2}}{\pi \lambda a} \approx -\frac{\rho}{\pi a} \ln \frac{r}{2a}$$

$\boxed{6.25}$




$$dR = \rho \frac{de}{ds} =$$

$$= \rho \frac{r\,d\theta}{2\pi r \sin\theta\, dr} = \frac{\rho\, d\theta}{2\pi \sin\theta\, dr}$$

$$d\Lambda = \frac{1}{dR}$$

$$d\Lambda_\theta = \int_0^a \frac{1}{dR} = \int_0^a 2\pi\lambda\, dr\, \frac{\sin\theta}{d\theta} = 2\pi a \lambda \frac{\sin\theta}{d\theta}$$

$$dR_\theta = \frac{1}{d\Lambda_\theta} \qquad R = \int_{\theta_0}^{\pi-\theta_0} dR_\theta = \frac{\rho}{2\pi a}\int_{\theta_0}^{\pi-\theta_0} \frac{d\theta}{\sin\theta} =$$

$$= \frac{\rho}{2\pi a} \ln \left| tg \frac{\theta_0}{2} \right| \Bigg|_{\theta_0}^{\pi-\theta_0} = \frac{-\rho}{\pi a} \ln \frac{r}{2a}$$

$\ln \frac{r}{2a} < 0$, т.к. $\frac{r}{2a} < 1$.